JHEP HYPER VERSION

Received: *June 1, 2001*, Accepted: *June 20, 2001*

Some remarks about Berkovits'superstring field theory

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Abstract: In this short note we would like to discuss general solutions of the Berkovits superstring field theory, in particular the string field action for fluctuation around such a solution. We will find that fluctuations obey the same equation of motion as the original field with the new BRST operator. Then we will argue that the superstring field theory action for fluctuation field has the same form as the original one.

Keywords: Superstrings and Heterotic Strings, D-branes.

JHEP06(2001)045

Contents

1. Introduction **1**

2. Review of superstring field theory **3**

3. Geometrical formulation of the superstring field theory action **4**

4. String field theory action for fluctuation around solution of the equation of motion **7**

5. Conclusion **10**

1. Introduction

In recent two years there was a great interest in a string field theory since this is only known nonperturbative definition of the string theory allowing off-shell computation of the tachyon potential (For review and extensive list of references, see [1], some recent works considering related problems are [10]–[23]. While bosonic string field theory [2] based on the Chern-Simons-like action is very well understood in superstring case the situation is not such a clear. The straightforward superstring extension of the bosonic string field theory [3] suffers from an emergence of contact divergences [4] that forces us to include higher-contact terms when two picture changing operators collide. (For discussion of this issue and other related problems we again recommend to see very good review [1].)

Another interesting approach to the problem of the construction of superstring field theory was given in Berkovits' works [6, 8]. While previous string field theories are based on the Chern-Simons-like actions, Berkovits formulation is based on the Wess-Zummino-Witten-like action. Two major advantages of Berkovits superstring field theory are that it is manifestly $\mathrm{SO}(3,1)$ super-Poincare invariant and that it does not require contact terms to remove tree-level divergences [24] (For recent review of Berkovits superstring field theory (BSFT), see [25].) In particular, it was shown that the calculation of the tachyon potential in BSFT is in a very good agreement with Sen's conjecture [26]. The tachyon kink and lump solution was also analysed (For recent work, see [14], for list of references, see [1].)

In this paper we will discuss BSFT from different point of view. As is well known from the bosonic case, when we expand the string field around any solution of the equation of motion, we find that the BRST operator has changed while the string field action has the same structure as the original one (for a nice discussion of this approach in case of closed bosonic string field theory, see, for example [7], for recent discussion, see [10, 16, 17]). We can then ask whether the similar process occurs in BSFT as well. In fact, analysis of this question could be helpful for extension of recent results [10, 16, 17] to the supersymmetric case and in the end could lead to more fundamental formulation of string theory and M theory.

In order to study these solutions, we firstly rewrite BSFT action in a slightly different form that resembles more similarity with the original Wess-Zumino-Witten (WZW) model (For review, see [27]). In this formulation we will regard two operators Q_B, η_0 as a part of generalised exterior derivative. Using this formulation we can manipulate with the action as with the ordinary WZW action using the language of differential geometry so that we are able very easily to find the variation of the action and hence an equation of motion.

As a next step we will analyse solution of the equation of motion arising from the variation of the action and fluctuation field around this solution. At first sight the action for fluctuation will appear to be different then the original one thanks to the presence of the field we expand around in it. In order to answer the question whether the fluctuation fields obey the same equation of motion and hence whether string field action is the same as the original one, we will not proceed in such a straightforward manner as in the bosonic string field theory where the new BRST operator naturally emerges from an action for fluctuating field. Rather we will study the equation of motion for fluctuating field and from it we will extract an information about a new BRST operator in BSFT. We will show on an example of the bosonic string field theory that this is an equivalent approach for searching of a new form of the BRST operator as the approach given above. Then we will show that in BSFT the BRST operator is modified with the background solution exactly in the same way as in the bosonic case. We believe that this is a nontrivial result since the form of the Berkovits superstring field theory is completely different then the open bosonic string theory. We also hope that our result could be helpful in the extension of the study of the vacuum string field theory [10], [16]–[20] to the supersymmetric case.

The plan of the paper is follows. In the next section 2 we briefly review BSFT theory.

In section 3 we rewrite the BSFT action in a slightly different form that resembles more similarity with the ordinary WSW model. Using this formalism we will be able to find a variation of the action and hence an equation of motion.

In section 4 we will discuss solution of the equation of motion arising from the variation of the action. Then we will study the behaviour of any string field around this new solution.

In conclusion 5 we will outline our results and suggest further extension of this work.

JHEP06(2001)045

2. Review of superstring field theory

In this section we would like to review basic facts about superstring field theory, for more details, see [1, 5, 6, 9]. The general off-shell string field configuration in the GSO(+) NS sector corresponding Grassmann even open string vertex operator Φ of ghost number 0 and picture number 0 in the combined conformal field theory of a $c = 15$ superconformal matter system, and b, c, β, γ ghost system with $c = -15$. We can also express β, γ in terms of ghost fields ξ, η, ϕ

$$\beta = \partial\xi e^{-\phi}, \ \gamma = \eta e^{\phi}, \tag{2.1}$$

the ghost number n_g and the picture number n_p assignments are as follows

$$\begin{aligned} b: n_g = -1, \quad n_p = 0 \qquad & c: \ n_g = 1, \quad n_p = 0; \\ e^{q\phi}: \ n_g = 0, \quad n_p = q; & \\ \xi: \ n_q = -1, \quad n_p = 1 \qquad & \eta: \ n_q = 1, \qquad n_p = -1. \end{aligned} \tag{2.2}$$

The BRST operator Q_B is given

$$Q_B = \oint dzj(z) = \int dz \left\{c(T_m + T_{\xi\eta} + T_\phi) + c\partial cb + \eta e^\phi G_m - \eta\partial\eta e^{2\phi} b\right\}, \tag{2.3}$$

where

$$T_{\xi\eta} = \partial\xi\eta, \ T_\phi = -\frac{1}{2}\partial\phi\partial\phi - \partial^2\phi, \tag{2.4}$$

T_m is a matter stress tensor and G_m is a matter superconformal generator. Throughout this paper we will be working in units $\alpha' = 1$.

The string field action is given [5, 6]

$$S = \frac{1}{2}\int \left((e^{-\Phi}Q_B e^{\Phi})(e^{-\Phi}\eta_0 e^{\Phi}) - \int_0^1 dt(e^{-t\Phi}\partial_t e^{t\Phi}) \left\{(e^{-t\Phi}Q_B e^{t\Phi}), (e^{-t\Phi}\eta_0 e^{t\Phi})\right\}\right), \tag{2.5}$$

where $\{A, B\} = AB + BA$ and $e^{-t\Phi}\partial_t e^{t\Phi} = \Phi$. Here the products and integral are defined by Witten's gluing prescription of the string. The exponential of string field is defined in the same manner $e^{\Phi} = 1 + \Phi + \frac{1}{2}\Phi \star \Phi + \cdots$. In the following we will not explicitly write $\star$ symbol. The basis properties of Q_B, η_0 which we will need in our analysis (for more details, see [1] and reference therein) are

$$\begin{aligned} Q_B^2 = 0, \quad \eta_0^2 = 0, \quad & \{Q_B, \eta_0\} = 0, \\ Q_B(\Phi_1\Phi_2) = & \ Q_B(\Phi_1)\Phi_2 + \Phi_1 Q_B(\Phi_2), \\ \eta_0(\Phi_1\Phi_2) = & \ \eta_0(\Phi_1)\Phi_2 + \Phi_1\eta_0(\Phi_2), \\ \int Q_B(\ldots) = 0, \qquad & \int \eta_0(\ldots) = 0, \end{aligned} \tag{2.6}$$

where Φ_1, Φ_2 are Grassmann even fields.

In the next section we rewrite (2.5) in the form that resembles more similarity with the original WZW model and that allows us to find very easily variation of the action.

JHEP06(2001)045

3. Geometrical formulation of the superstring field theory action

In this section we rewrite the BSFT action (2.5) in a geometrical formalism which will allow us to find very easily variation of the action [5, 6] and consequently equation of motion.

Let us define generalised exterior derivative as follows

$$\begin{aligned}
dX &= \partial_t X dt + Q_B(X)dx^2 + \eta_0(X)dx^3 = \partial_i X dx^i\,, \\
dx^2\wedge dx^3 &= dx^3\wedge dx^2\,, \quad dx^1\wedge dx^2 = -dx^2\wedge dx^1\,, \quad dx^1\wedge dx^3 = -dx^3\wedge dx^1\,, \\
dx^2\wedge dx^2 &= dx^3\wedge dx^3 = 0\,,
\end{aligned} \tag{3.1}$$

with any string field X. Now we prove the nilpotence of this operator $d^2X = 0$. Then we have

$$d^2(X) = d(\partial_i(X)dx^i) = \partial_i(\partial_j X)dx^i\wedge dx^j = 0\,, \tag{3.2}$$

since for $i = 1, j = 2,3$ operators Q_B, η_0, ∂_t commute but $dx^1\wedge dx^{2,3} = -dx^{2,3}\wedge dx^1$ and for $i,j = 2,3$ operators anticommute (note $\{Q,\eta_0\} = 0$) but $dx^1\wedge dx^2 = dx^2\wedge dx^1$ commute among themselves. We have also used $Q_B^2 = \partial_2(\partial_2) = 0,\ \eta_0^2 = \partial_3(\partial_3) = 0$. Let us turn to the rewriting the action (2.5). In the following we introduce a notation [9]

$$G = e^{\Phi}\,,\ \hat{G} = e^{t\Phi}\,. \tag{3.3}$$

We start our analysis with the second term in (2.5) which we will write as

$$\frac{1}{3}\int \hat{G}^{-1}d\hat{G}\wedge\hat{G}^{-1}d\hat{G}\wedge\hat{G}^{-1}d\hat{G}\,, \tag{3.4}$$

where now the integral denotes integration over t and also abstract Witten's string field integration [2]. We implicitly presume that this integration is defined as

$$\int\int dt\,\{\dots\} = \int \{\dots\}\,dx^1\wedge dx^2\wedge dx^3\,. \tag{3.5}$$

We also see that $\hat{G}(t)$ is a function of t with the property that for $t = 1, \hat{G}(1) = G,\ \hat{G}(0) = 1$. We will show that the formulation (3.4) corresponds to the second term in (2.5)

$$\begin{aligned}
&\frac{1}{3}\int \hat{G}^{-1}\partial_i\hat{G}\hat{G}^{-1}\partial_j\hat{G}\hat{G}^{-1}\partial_k\hat{G}dx^i\wedge dx^j\wedge dx^k = \\
&\quad = \frac{1}{3}\int \hat{G}^{-1}\partial_1\hat{G}[\hat{G}^{-1}\partial_2\hat{G}\hat{G}^{-1}\partial_3\hat{G} + \hat{G}^{-1}\partial_3\hat{G}\hat{G}^{-1}\partial_2\hat{G}]dx^1\wedge dx^2\wedge dx^3 + \\
&\qquad + \hat{G}^{-1}\partial_2\hat{G}[\hat{G}^{-1}\partial_3\hat{G}\hat{G}^{-1}\partial_1\hat{G} - \hat{G}^{-1}\partial_1\hat{G}\hat{G}^{-1}\partial_3\hat{G}]dx^1\wedge dx^2\wedge dx^3 + \\
&\qquad + \hat{G}^{-1}\partial_3\hat{G}[\hat{G}^{-1}\partial_2\hat{G}\hat{G}^{-1}\partial_1\hat{G} - \hat{G}^{-1}\partial_1\hat{G}\hat{G}^{-1}\partial_2\hat{G}]dx^1\wedge dx^2\wedge dx^3 = \\
&\quad = \int \hat{G}^{-1}\partial_1\hat{G}[\hat{G}^{-1}\partial_2\hat{G}\hat{G}^{-1}\partial_3\hat{G} + \hat{G}^{-1}\partial_3\hat{G}\hat{G}^{-1}\partial_2\hat{G}]dx^1\wedge dx^2\wedge dx^3\,,
\end{aligned} \tag{3.6}$$

which we wanted to prove. In the previous expression we have used (we omit the symbol $dx^1 \wedge dx^2 \wedge dx^3$ which does not change in the following manipulation)

$$
\begin{aligned}
\int \hat{G}^{-1}\partial_2\hat{G}\hat{G}^{-1}\partial_3\hat{G}\hat{G}^{-1}\partial_1\hat{G} &= \int \hat{G}^{-1}\partial_1\hat{G}\hat{G}^{-1}\partial_2\hat{G}\hat{G}^{-1}\partial_3\hat{G}\,, \\
-\int \hat{G}^{-1}\partial_2\hat{G}\hat{G}^{-1}\partial_1\hat{G}\hat{G}^{-1}\partial_3\hat{G} &= \int \hat{G}^{-1}\partial_1\hat{G}\hat{G}^{-1}\partial_3\hat{G}\hat{G}^{-1}\partial_2\hat{G}\,, \\
\int \hat{G}^{-1}\partial_3\hat{G}\hat{G}^{-1}\partial_2\hat{G}\hat{G}^{-1}\partial_1\hat{G} &= \int \hat{G}^{-1}\partial_1\hat{G}\hat{G}^{-1}\partial_3\hat{G}\hat{G}^{-1}\partial_2\hat{G}\,, \\
-\int \hat{G}^{-1}\partial_3\hat{G}\hat{G}^{-1}\partial_1\hat{G}\hat{G}^{-1}\partial_2\hat{G} &= \int \hat{G}^{-1}\partial_1\hat{G}\hat{G}^{-1}\partial_2\hat{G}\hat{G}^{-1}\partial_3\hat{G}\,,
\end{aligned}
\tag{3.7}
$$

which follows from the definition of Q_B, η_0 operators. Using description (3.4) we can easily find the variation of the WZW term

$$
\begin{aligned}
\delta\frac{1}{3}\int(\hat{G}^{-1}d\hat{G})^3 &= \int \delta\hat{G}^{-1}d\hat{G}\wedge(\hat{G}^{-1}d\hat{G})^2 + \int \hat{G}^{-1}d\delta\hat{G}\wedge(\hat{G}^{-1}d\hat{G})^2 \\
&= -\int \hat{G}^{-1}\delta\hat{G}\hat{G}^{-1}d\hat{G}\wedge(\hat{G}^{-1}d\hat{G})^2 + \int d(\delta\hat{G}(\hat{G}^{-1}d\hat{G})^2\hat{G}^{-1}) - \\
&\quad -\int \delta\hat{G}d((\hat{G}^{-1}d\hat{G})^2\hat{G}^{-1}) = \int d[\delta\hat{G}(\hat{G}^{-1}d\hat{G})^2\hat{G}^{-1}]\,,
\end{aligned}
\tag{3.8}
$$

where we have used

$$
-\int \delta\hat{G}d((\hat{G}^{-1}d\hat{G})^2\hat{G}^{-1}) = \int \hat{G}^{-1}\delta\hat{G}(\hat{G}^{-1}d\hat{G})^3\,. \tag{3.9}
$$

We have also used the fact that the exterior derivative acts on various forms as usual exterior derivative. More precisely, let $\omega = \omega_i dx^i, \eta = \eta_i dx^i$ are one forms with Grassmann odd $\omega_{2,3},\ \eta_{2,3}$ components. Then we have

$$
\begin{aligned}
d(\omega\wedge\eta) &= \partial_k(\omega_i\eta_j)dx^k\wedge dx^i\wedge dx^j \\
&= (\partial_k\omega_i dx^k\wedge dx^i)\eta_j\wedge dx^j - \omega_i dx^i\wedge(\partial_k\eta_j dx^k\wedge dx^j) \\
&= d\omega\wedge\eta - \omega\wedge d\eta\,,
\end{aligned}
\tag{3.10}
$$

where the minus sign emerges either from the anticommutative nature of $dx^1 \wedge dx^{2,3}$ and the presence of the ordinary derivative ∂_1 or through the anticommutative relation between the derivatives $\partial_{2,3}$ and ω_{23}, η_{23} and commutative nature of $dx^{2,3}$. Then the variation of the WZW term is given (Remember, that in our abstract interpretation the boundary of the "space" we integrate over is in the points $t = 0, 1$)

$$
\int \hat{G}^{-1}\delta\hat{G}(\hat{G}^{-1}d\hat{G})^2|_{t=0}^{t=1}\,, \tag{3.11}
$$

where the integration corresponds to Witten's string field integration or in our notation to the integration over abstract space spanned with $x^{2,3}$ coordinates. The fact that there is not "boundary" of the space labelled with coordinates x^2, x^3 can be also seen from the definition $\int Q_B(\ldots) = \int \eta_0(\ldots) = 0$. Since we know that $\hat{G}(t)$ is a function of t with the property $\hat{G}(1) = G, \hat{G}(0) = 1$ the variation of the action is equal to

$$\int G^{-1}\delta G(G^{-1}Q_B(G)G^{-1}\eta_0(G) + G^{-1}\eta_0(G)G^{-1}Q_B(G)) \tag{3.12}$$

and using the fact that the expression in the bracket can be written as

$$\begin{aligned}
-(\eta_0(G^{-1}Q_B(G)) + Q_B(G^{-1}\eta_0(G))) &= -\eta_0(G^{-1})Q_B(G) - G^{-1}\eta_0(Q_B(G)) - \\
&\quad -Q_B(G^{-1})\eta_0(G) - G^{-1}Q_B(\eta_0(G)) \\
&= G^{-1}\eta_0(G)G^{-1}Q_B(G) + G^{-1}Q_B(G)G^{-1}\eta_0(G)
\end{aligned}$$

so that the variation of the WZW term is equal to

$$\delta S_{WZW} = -\int G^{-1}\delta G\left(\eta_0(G^{-1}Q_B(G)) + Q_B(G^{-1}\eta_0(G))\right). \tag{3.13}$$

In the similar way we rewrite the first term in (2.5). In order to do it we should introduce the operation of the Hodge dual $*$. Note that the integration in the first term in (2.5) is performed on the space spanned with $x^{2,3}$, so we can define the Hodge dual operation for this space as follows

$$\begin{aligned}
*(dx^i) &= \epsilon^i_j dx^j = g^{ik}\epsilon_{kj}dx^j, \qquad \epsilon_{23} = -\epsilon_{32} = 1, \\
(dx^2) &= g^{23}\epsilon_{32}dx^2 = -dx^2, \quad *(dx^3) = g^{32}\epsilon_{23}dx^3 = dx^3,
\end{aligned}$$

where g^{ij} is an auxiliary metric with nonzero components $g^{23} = g^{32} = 1$. Then we claim that the first term in (2.5) can be written as

$$\frac{1}{2}\int G^{-1}dG \wedge *G^{-1}dG, \tag{3.14}$$

since we have[1]

$$*G^{-1}dG = G^{-1}\partial_2 G * (dx^2) + G^{-1}\partial_3 G * (dx^3) = -G^{-1}\partial_2 G dx^2 + G^{-1}\partial_3 G dx^3 \tag{3.15}$$

and then the kinetic term is equal to

$$\begin{aligned}
&\frac{1}{2}\int (G^{-1}\partial_2 G dx^2 + G^{-1}\partial_3 G dx^3) \wedge (-G^{-1}\partial_2 G dx^2 + G^{-1}\partial_3 G dx^3) = \\
&\quad = \frac{1}{2}\int \left(G^{-1}\partial_2 G G^{-1}\partial_3 G dx^2 \wedge dx^3 - G^{-1}\partial_3 G G^{-1}\partial_2 G dx^2 \wedge dx^3\right) \\
&\quad = \int G^{-1}\partial_2 G G^{-1}\partial_3 G dx^2 \wedge dx^3 \\
&\quad = \int G^{-1}Q_B(G)G^{-1}\eta_0(G)
\end{aligned} \tag{3.16}$$

[1] In this kinetic term the exterior derivative does not contain ∂_t derivative. Consequently the integration $\int d\{\ldots\}$ is equal to zero thanks to the definition of Q_B, η_0.

JHEP06(2001)045

with the variation

$$
\begin{aligned}
\delta\frac{1}{2}\int G^{-1}dG\wedge *G^{-1}dG &= \int \delta G^{-1}dG\wedge *G^{-1}dG+\int G^{-1}d\delta G\wedge *G^{-1}dG \\
&= -\int G^{-1}\delta G G^{-1}dG\wedge *G^{-1}dG- \\
&\quad -\int \delta G d*(G^{-1}dG)G^{-1}-\int dG^{-1}\delta G\wedge *G^{-1}dG \\
&= -\int G^{-1}\delta G d\left(G^{-1}*dG\right) \qquad (3.17) \\
&= \int G^{-1}\delta G(\eta_0(G^{-1}Q_B(G))-Q_B(G^{-1}\eta_0(G)))dx^2\wedge dx^3\,.
\end{aligned}
$$

Collecting all previous results (3.12), (3.13), (3.17) we obtain

$$
\delta S=-G^{-1}\delta G\left\{d\left(G^{-1}*dG\right)+G^{-1}dG\wedge G^{-1}dG\right\}=G^{-1}\delta G\eta_0(G^{-1}Q_B(G)) \qquad (3.18)
$$

so that the equation of motion has a form

$$
d\left(G^{-1}*dG\right)+G^{-1}dG\wedge G^{-1}dG=0\Rightarrow \eta_0(G^{-1}Q_B(G))=0\,, \qquad (3.19)
$$

with agreement with [5, 6, 9]. It is clear that the upper result is not new but we believe that our auxiliary "geometrical formulation" could be helpful for better understanding of properties of the BSFT action.

In the next section we use this result and we will study the solution of the equation of motion given above (3.19).

4. String field theory action for fluctuation around solution of the equation of motion

In this section we will discuss the string field theory around any background configuration which is a solution of (3.19). Since we will not perform any explicit calculation we will again use abstract Witten's formalism in string field theory [2]. As usual we will not write explicitly the string field theory star product $\star$. Let us consider any string field Φ_0, corresponding to $G_0=e^{\Phi_0}$, which is a solution of the equation of motion (3.19)

$$
\eta_0(G_0^{-1}Q_B(G))=0\,. \qquad (4.1)
$$

Now we would like to study the fluctuation around this solution. For that reason we write general string field containing fluctuation around this solution as

$$
G=G_0h\,,\qquad h=e^{\phi}\,,\qquad G^{-1}=h^{-1}G_0^{-1}\,. \qquad (4.2)
$$

To see that this field really describes fluctuations around solution G_0 note that for $\phi=0, G=G_0$. It is also clear that any string field in the form $e^{\Phi_0+\phi'}$ can be always rewritten in the form given above.

Inserting this upper expression in (2.5) we obtain an action for ϕ

$$
\begin{aligned}
S = \frac{1}{2}\int h^{-1}G_0^{-1}d(G_0h) * h^{-1}G_0^{-1}d(G_0h) - \\
-\frac{1}{3}\int \hat{h}^{-1}\hat{G}_0^{-1}d(\hat{h}\hat{G}_0) \wedge \hat{h}^{-1}\hat{G}_0^{-1}d(\hat{h}\hat{G}_0) \wedge \hat{h}^{-1}\hat{G}_0^{-1}d(\hat{h}\hat{G}_0)\,.
\end{aligned}
\tag{4.3}
$$

Now we would like to ask the question what form of the equation of motion obeys shifted field $h = e^{\phi}$. We will show on an example of the bosonic string field theory that this is a meaningful approach for extracting an information about a possible new BRST operator. In particular, it is well known that the WZW term cannot be expressed in any closed form (For review, see [27].) as an integral over two dimensional space spanned with x^2, x^3 so it would be difficult to extract the form of the new BRST operator directly from (4.3). On the other hand, as we have seen in the previous section, the variation of the action can be written as an integral over $x^{2,3}$ so that it is seems to be more efficient to gain information about new Q_B, η_0 from the equation of motion.

In order to obtain this equation we will vary (4.3) with respect to $G = G_0h$, $\delta G = G_0\delta h$, $\delta h = G_0^{-1}\delta G$ so we get

$$
d(h^{-1}G_0^{-1} * d(G_0h)) + G_0^{-1}h^{-1}d(G_0h) \wedge h^{-1}G_0^{-1}d(G_0h) = 0\,, \tag{4.4}
$$

or equivalently

$$
\eta_0(h^{-1}G_0^{-1}Q_B(G_0h)) = \eta_0(h^{-1}Q_B(h) + h^{-1}G_0^{-1}Q_B(G_0)h) = 0\,. \tag{4.5}
$$

We can add to the upper expression the term

$$
-\eta_0(G_0^{-1}Q_B(G_0)) = \eta_0(h^{-1}h(-1)^hG_0^{-1}Q_B(G_0)) \tag{4.6}
$$

that is equal to zero according to (4.1). We have chosen minus sign in the upper expression since possible new BRST operator should obey all string field theory axioms (2.6) and as we will see minus sign is crucial for it. Then we get the final result

$$
\begin{aligned}
&\eta_0(h^{-1}Q_B(h) + h^{-1}Ah - h^{-1}hA)) = \eta_0(h^{-1}\tilde{Q}_B(h)) = 0\,, \\
&\tilde{Q}_B(X) = Q_B(X) + AX - (-1)^X XA\ , \forall X,\ A = e^{-\Phi_0}Q_B(e^{\Phi_0})\,.
\end{aligned}
$$

Upper expression suggests that the BRST operator has changed through the solution of the equation of motion. Note that A has a correct properties to be added to the BRST operator since has a ghost number one and picture number zero. We also see that η_0 does not change which from our point of view seems to be natural result since η_0 corresponds to the bosonisation of the ghost fields so that it should not depend on any background configuration.

Let us compare this situation with Witten's bosonic open string field theory where the string field action is [2]

$$S = \frac{1}{2}\int \Psi Q_B \Psi + \frac{1}{3}\int \Psi\Psi\Psi\,. \qquad (4.7)$$

Let Ψ_0 be a solution of the equation of motion

$$Q_B\Psi_0 + \Psi_0\Psi_0 = 0 \qquad (4.8)$$

and let any string field Ψ containing fluctuation around this solution has a form $\Psi = \Psi_0 + \psi$. When we insert this string field into (4.7) and perform variation we get an equation of motion for ψ

$$Q_B\psi + \psi\Psi_0 + \Psi_0\psi + \psi\psi + (Q_B\Psi_0 + \Psi_0\Psi_0) = 0 \Rightarrow Q'_B\psi + \psi\psi = 0\,, \qquad (4.9)$$

where the new BRST operator Q'_B is defined as

$$Q'_B(X) = Q_B X + \Psi_0 X - (-1)^X X\Psi_0\,, \qquad (4.10)$$

with X labels grading of the string field X. In other words, the BRST operator Q'_B acting on a field ψ is not the same as the original one which is certainly natural result since the field ψ propagates around different background then original string field Ψ. We also see that the bosonic string field action is the same as the original one as it should be since its form should not depend on any particular background CFT. We must also say that we could obtain the form of the new BRST operator Q'_B directly from the action (4.7) without writing the equation of motion for ϕ as is well known for a long time. We have chosen the second approach which gives an equivalent result in order to see direct relation with the analysis performed in case of BSFT.

From this example we see a striking similarity with Berkovits string field theory even if it seems to be difficult to see that the action (4.3) has the same form as the original one (2.5), however with the different BRST operator $\tilde{Q}_B$. We determine this action using the fact that the equation of motion for ϕ are the same as the equation of motion obtained from (2.5) so it is natural that it arises from an action with the same form as (2.5) and with the BRST operator $\tilde{Q}_B$. Hence the action for fluctuation is

$$S(\phi) = \frac{1}{2}\int e^{-\phi}\tilde{Q}_B(e^{\phi})e^{-\phi}\eta_0(e^{\phi}) - \int_0^1 dte^{-t\phi}\partial_t e^{t\phi}\left\{e^{-t\phi}\tilde{Q}_B(e^{t\phi}), e^{-t\phi}\eta_0(e^{t\phi})\right\}. \qquad (4.11)$$

Note that in the upper expression we have shifted the action so that does not contain constant part $S(G_0)$.

To finish our analysis we should prove that the BRST operator $\tilde{Q}_B$ obeys all string field theory axioms. We firstly prove its nilpotence

$$\begin{aligned}\tilde{Q}_B^2(X) &= \tilde{Q}_B(Q_B(X) + AX - (-1)^X XB) \\ &= Q_B(Q_B(X) + AX - (-1)^X XA) + +A(Q_B(X) + AX - (-1)^X XA) - \\ &\quad -(-1)^{X+1}(Q_B(X) + AX - (-1)^X XA)A \\ &= (Q_B(A) + AA)X - X(Q_B(A) + AA) = 0\,, \end{aligned} \tag{4.12}$$

where we have used

$$Q_B(A) = Q_B(G_0^{-1}Q_B(G_0)) = -G_0^{-1}Q_B(G_0)G_0^{-1}Q_B(G_0) = -AA\,. \tag{4.13}$$

We can also see that

$$\begin{aligned}\tilde{Q}_B(XY) &= Q_B(XY) + AXY - (-1)^{X+Y}XYA \\ &= (Q_B(X) + AX - (-1)^X XA)Y + (-1)^X X(Q_B(Y) + +AY - (-1)^Y YA) \\ &= \tilde{Q}_B(X)Y + (-1)^X \tilde{Q}_B(Y) \end{aligned} \tag{4.14}$$

and

$$\{\eta_0, \tilde{Q}_B\}(X) = \eta_0(AX - (-1)^X XA) + A\eta_0(X) - (-1)^{X+1}\eta_0(X)A = 0\,, \tag{4.15}$$

since $\eta_0(A) = 0$ as follows from (4.1) and consequently $\eta_0(AX) = -A\eta_0(X)$. And it is also easy to see that

$$\int \tilde{Q}_B(X) = 0\,. \tag{4.16}$$

We must say a few words about this result. It is interesting to see that the nilpotence of the new BRST operator $\tilde{Q}_B$ does not depend on the fact that G_0 is a solution of the equation of motion which is a difference with the bosonic string field theory where the nilpotence of the new BRST operator depends on the fact that Ψ_0 is the solution of the string field theory equation of motion. On the other hand we have seen that the requirement, that the anticommutator $\{\tilde{Q}_B, \eta_0\}$ must be equal to zero, can be obeyed only when G_0 solves (4.1). In other words, the requirement that Φ_0 is a solution of the equation of motion is crucial in BSFT as well, however from different reason then in the bosonic string field theory.

5. Conclusion

The main goal of this paper was to study the fluctuation around any solution of the Berkovits superstring field theory [5, 6, 25]. For that reason we have rewritten the action into slightly different form which allows us to find very easily its variation and hence an equation of motion. Even if this formulation is certainly not new, in the context of the superstring field theory could give better sight how the variation and hence string field theory equation of motion arises from (2.5).

Then we have studied solution of the equation of motion obtained from the BSFT action. We have been mainly concerned with the question of the form of the action for fluctuation around this solution. We have argued that thanks to the nontrivial form of the BSFT action it is difficult to see directly from it a form of a new BRST operator. Then we have shown on an example of the bosonic string field theory that we can obtain an information about the BRST operator from an equation of motion. When we have applied this method to the BSFT case we have seen that the BRST operator changes in the similar way as in the bosonic case as we could expect. Then we have shown that the new BRST operator obeys all axioms of the BSFT.

We believe that the results presented in this paper are nontrivial. In particular, we hope that they could be helpful in the extension of the recent interesting works considering vacuum string field theory [10, 16, 17, 18, 20] to the supersymmetric case that hopefully in the end could give a new insight into the structure of the superstring theory.

Acknowledgments

We would like to thank Rikard von Unge for very useful discussions and especially Ashoke Sen for correspondence about this subject.

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